This sheet will be discussed in the exercise class on 4 December. You are welcome to submit your solutions at the end of the exercise class or anytime earlier.

**Exercise 9.1. Example: understanding the simplified group law.**

1. Show that \([0 : 1 : 0]\) is an inflection point of \(C\) in the simplified group law 9.4.
2. In the simplified group law 9.4, explain briefly how to find all points \(P \in C\) such that \(P + P = O\).
3. Consider the curve and the group law in Example 9.6. Let \(A = [2 : 1 : 1]\) and \(B = [-2 : -1 : 1]\). Use the simplified group law to find out \(-A\), \(-B\) and \(A + B\).

**Exercise 9.2. Example of group law.** Consider the non-singular cubic curve \(C = \mathbb{V}(y^2z - x^3 - 4xz^2) \subseteq \mathbb{P}^2\). Let \(O = [0 : 1 : 0]\) be the identity element in the group law.

1. Find all points where \(C\) meets the line \(L_1 = \mathbb{V}(z)\) and specify their multiplicities. Do the same for the lines \(L_2 = \mathbb{V}(x)\) and \(L_3 = \mathbb{V}(y - 2x)\).
2. Find the order of the subgroup generated by the point \(P = [2 : 4 : 1] \in C\).
3. Find all points \(Q \in C\) such that \(Q + Q = O\).

**Exercise 9.3. Example: Tate’s normal form.** Consider the projective closure \(C\) of the cubic curve \(C_2 = \mathbb{V}(y^2 + sxy - ty - x^3 + tx^2) \subseteq \mathbb{A}^2\) for some fixed \(s, t \in \mathbb{k}\) where \(t \neq 0\). Assume \(C\) is non-singular. Let the point at infinity \(O = [0 : 1 : 0]\) be the identity element in the group law on \(C\).

1. For any point \(P = (a, b) \in C_2\), show that \(-P = (a, -b - sa + t)\) in the group law.
2. Suppose \(Q = (0, 0) \in C_2\). Show that \(Q + Q = (t, t(1 - s))\) in the group law.

**Exercise 9.4. Pascal’s mystic hexagon.** Let \(X \subseteq \mathbb{P}^2\) be an irreducible conic. Let \(ABCDEF\) be a hexagon whose vertices are inscribed in \(X\). Assume the three pairs of opposite sides meet in points \(P, Q, R\) respectively. (To be precise, the lines \(FA\) and \(CD\) meet at \(P\); the lines \(AB\) and \(DE\) meet at \(Q\); the lines \(BC\) and \(EF\) meet at \(R\).) Show that \(P, Q, R\) are collinear. (That means, the three points are on the same line in \(\mathbb{P}^2\).) You can follow these steps (the idea is already used in the proof of Proposition 9.13):

1. Sketch a picture to illustrate the given situation.
2. The three lines \(FA, BC\) and \(DE\) form a cubic curve \(C_1\); the three lines \(AB, CD\) and \(EF\) form a cubic curve \(C_2\). Find \(C_1 \cap C_2\).
3. Consider a third cubic \(C_3\) given by the union of the conic \(X\) and the line \(PQ\). Then apply Lemma 9.12.