BRIEF SUMMARY OF LECTURE 11

If you found the lecture on Monday 02/11 a little/somewhat/very/extremely obscure, I hope this summary helps you understand it better.

We discussed 2 operations on polynomials (dehomogenisation and homogenisation) and 2 operations on algebraic sets (standard affine piece and projective closure) on Monday.

Operations on polynomials

We can *dehomogenise* a homogeneous polynomial $f(z_0, z_1, \cdots, z_n)$ simply by choosing one variable z_i and setting $z_i = 1$. For example: the dehomogenisation of f with respect to z_0 is the polynomial $f(1, z_1, \cdots, z_n)$. Notice that the resulting polynomial is not homogenous in general, and has one fewer variable than the original polynomial.

We can *homogenise* any polynomial $g(z_1, \cdots, z_n)$ using an extra variable. If

$$g = g_0 + g_1 + \cdots + g_{d-1} + g_d$$

is the homogeneous decomposition of g, where $d = \deg g$, then its homogenisation with respect to z_0 is

$$\overline{g} = z_0^d \cdot g_0 + z_0^{d-1} \cdot g_1 + \cdots + z_0 \cdot g_{d-1} + g_d.$$

Notice that $\overline{g}$ is a homogeneous polynomial, and has one more variable than the original polynomial g. Sometimes we write the formula for $\overline{g}$ in a different form

$$\overline{g} = z_0^d \cdot g\left(\frac{z_1}{z_0}, \cdots, \frac{z_n}{z_0}\right).$$

The advantage of this form is that we need not use the homogeneous decomposition of g. It is not difficult to check the two formulas give the same $\overline{g}$. You can try this in explicit examples.

Operations on algebraic sets

A projective algebraic set $X \subseteq \mathbb{P}^n$ has *standard affine pieces* $X_i = X \cap U_i$ for $i = 0, 1, \cdots, n$, which are affine algebraic sets. To obtain each X_i, one only needs to dehomogenise the defining equations of X by setting $z_i = 1$.

An affine algebraic set $X \subseteq \mathbb{A}^n$ has a *projective closure* $\overline{X} \subseteq \mathbb{P}^n$, which is a projective algebraic set. A special case: if $X = \mathbb{V}_a(f)$ is defined by one polynomial f, then $\overline{X} = \mathbb{V}_p(\overline{f})$ is defined by the homogenisation $\overline{f}$ of f. The general case: if X is defined by more than one polynomial, one has to homogenise all polynomials $f \in \mathbb{I}_a(X)$; see Definition 6.5. This definition is impractical for doing any explicit calculation.

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