EXTRA HINTS FOR EXERCISE SHEET 1

To give you a headstart with the first exercise sheet, here are some extra hints that you may find helpful. Always try to work on the problems without reading the hints first (as hints are rarely given in exams). If you are still stuck after reading the hints, have a discussion with your classmates, and feel free to ask me for more clues.

Exercise 1.1. There are many possible answers to each part. Just make sure that points in X are ALL common solutions of polynomials in S. For instance, $S = \{y - x\}$ is NOT a correct answer to part (2), because the polynomial $y - x$ has many more other solutions which are not in X.

Exercise 1.2. For (1), you need to show that each point p in $\mathbb{V}(S_1)$ is also in $\mathbb{V}(S_2)$. So you can probably start your proof with: "given any point $p \in \mathbb{V}(S_1)$, we have $f(p) = 0$ for every polynomial $f \in S_1$. Then ...". A brief hint for (2) was given in the lecture. For (3), you need to prove mutual inclusions. For one direction $\cap_\alpha(\mathbb{V}(S_\alpha)) \subseteq \mathbb{V}(\cup_\alpha S_\alpha)$, you can probably start your proof with: "give any point $p \in \cap_\alpha(\mathbb{V}(S_\alpha))$, we have $p \in \mathbb{V}(S_\alpha)$ for every α. Then ...". The other direction $\cap_\alpha(\mathbb{V}(S_\alpha)) \supseteq \mathbb{V}(\cup_\alpha S_\alpha)$ can be proved in a similar way. Notice that $\cup_\alpha S_\alpha$ is the union of all S_α's. Namely, it is a big collection of all polynomials from any of S_α's. Part (4) can be done in a similar way to part (3).

Exercise 1.3. This interesting problem has been broken into three easy parts. The only tricky point is in part (2). Say $X = \mathbb{V}(S)$ is an algebraic set in $\mathbb{A}^1$. If S does not contain any non-zero polynomial, then $X = \mathbb{A}^1$. Otherwise, there is some $f \in S$ which is a non-zero polynomial. Every point in X should be a solution of f. Then you can use the hint given in the problem.

Exercise 1.4. Notice that for an ideal $J \subseteq R/I$, $q^{-1}(J) = \{r \in R \mid r + I \in J\}$. This expression could be useful in parts (1) and (2). For part (3), you can use the results in the first two parts to get an ascending chain of ideals $q^{-1}(J_1) \subseteq q^{-1}(J_2) \subseteq q^{-1}(J_3) \subseteq \cdots$ in R. The fact that R is a Noetherian ring should be helpful at this point. Part (4) is straightforward once you establish part (3).

Date: October 2, 2015.