EXTRA HINTS FOR EXERCISE SHEET 3

Exercise 3.1. This simple example should help you get familiar with the terminologies. Most parts are straightforward. In part (2), to prove $G(f)$ is an algebraic set, you need to find polynomials that define it, which consist of polynomials that define X and an extra one. In part (4), the inverse map is given by a certain projection map as introduced in part (1). In part (5), you need to explain what X and f are in that specific example.

Exercise 3.2. Part (1) is easy. In part (2), you need to show that every point $(x, y) \in X$ can be written as $\varphi(t)$ for some $t \in \mathbb{A}^1$, but the point $(0, 0)$ can be written as $\varphi(t)$ for two different values of t. More precisely, when $x = 0$, one can find $(x, y) = f(1) = f(-1)$; when $x \neq 0$, one can find $(x, y) = f(t)$ for $t = y/x$. For part (3), to show the polynomial is irreducible, you can simply modify the argument in Example 1 in the algebra review session appropriately.

Exercise 3.3. Parts (1) and (3) are similar to those in Exercise 3.2. In part (2), you need to show that every point $(x, y) \in X$ can be written as $\varphi(t)$ for a unique $t \in \mathbb{A}^1$. It is still helpful to distinguish the two cases: $x = 0$ and $x \neq 0$. In part (4), first find the image $\varphi^*(f)$ for any polynomial $f(x, y)$. Then you need to argue that $\varphi^*(f)$ never contains a term which has degree 1 in t. This implies that the polynomial $t \in \Bbbk[t] = \Bbbk[\mathbb{A}^1]$ is not in the image of φ^* (or indeed, every polynomial in $\Bbbk[t]$ with a non-zero term of degree 1 is not in the image of φ^*).

Exercise 3.4. In part (1), the inverse map is a certain projection map. For part (2), you have essentially done the work in Exercise 2.4 (2)(3).

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