EXTRA HINTS FOR EXERCISE SHEET 10

Exercise 10.1. For part (1), Definition 10.8 and Remark 10.9 imply that the given algebraic set is a line in $\mathbb{P}^2$. It is clear that every point on the line is a point in P, hence it is a line in P. For part (2), you can use a similar argument as in Exercise 4.3 (2).

Exercise 10.2. Part (1) is similar to the previous question. You can use Definition 10.8 and Remark 10.9 to prove it is a line in $\mathbb{P}^2$. Then you need to use the defining equation of Q to check that every point on the line is a point in Q. For part (2), Consider two lines $L = \mathbb{V}(az_0 + bz_1, az_2 + bz_3)$ and $L' = \mathbb{V}(a'z_0 + b'z_1, a'z_2 + b'z_3)$ where $[a : b]$ and $[a' : b']$ are two different points in $\mathbb{P}^1$. You need to prove that the system of four equations that define L and L' has no non-zero solution, therefore L and L' have no common point. For part (3), first of all we need to show that every point in Q lies on at least one of the lines. Indeed, given a point $[z_0 : z_1 : z_2 : z_3] \in Q$, we can choose $[a : b] = [z_1 : -z_0]$ or $[z_3 : -z_2]$. At least one of the choices makes sense because not all coordinates are zero. Then we need to show each point in Q lies on at most one of the lines, which is an easy consequence of part (2). Part (4) is interesting. The family of lines you write down should be very similar to the ones given in part (1).

Exercise 10.3. Part (1): it is a standard calculation. Just remember not to expand the round brackets. Part (2): also a standard calculation. It is also very helpful not to expand the round brackets.

Exercise 10.4. Enjoy your holidays!

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