EXTRA HINTS FOR EXERCISE SHEET 7

Exercise 7.1. By the definition of discriminants, you need to compute some traces first, then compute the discriminant. Exercise 6.2 should be helpful, but you still have some more traces to compute. But as long as you are patient, you can do this problem for sure!

Exercise 7.2. For part (1), since $\beta_1, \beta_2, \cdots, \beta_n$ is an integral basis for I, every element α_i can be written as an integral linear combination of $\beta_1, \beta_2, \cdots, \beta_n$. Let M be the transition matrix. Use Proposition 7.6 to get the first equation. The second equation is similar, based on the fact that $\alpha_1, \alpha_2, \cdots, \alpha_n$ is an integral basis for I. For part (2), what can you get if you combine the two equations in part (1) using substitution?

Exercise 7.3. The formula in Example 6.18 makes the calculation of traces straightforward. By Proposition 7.2, 1 and ω form an integral basis for $\mathcal{O}_K$, so the discriminants in question is precisely Δ_K.

Exercise 7.4. Part (1) uses the definition of an ideal. Part (2) can be proved by using the definition of linear dependence. For part (3), since $I = (\alpha)$, every element $\gamma \in I$ can be written as $\gamma = \alpha\beta$ for some $\beta \in \mathcal{O}_K$. Then we can write β as an integral linear combination of $\omega_1, \omega_2, \cdots, \omega_n$. For part (4), remember that an integral basis for $\mathcal{O}_K$ is given by 1 and $\omega = \sqrt{3}$.

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