# EXTRA HINTS FOR EXERCISE SHEET 8

**Exercise 8.1.** In both parts we need to find the norm of a principal ideal. By Proposition 8.9, we need to compute the norm of the generator. This is clear for part (1). Part (2) is also straightforward. You can either write down the matrix corresponding to $L_a$, or use some properties proved in last exercise sheet.

**Exercise 8.2.** This exercise gives you some practice on the sum and product of ideals. The first three parts can be proved by using the definitions of the sum and product of ideals. In part (4), the ideal $(x, y)$ is defined to be the sum of two principal ideals, hence every element in $(x, y)$ can be written as $xf + yg$ for some $f, g \in \mathbb{k}[x, y]$. You need to show both directions: every element in $(x, y)$ is a polynomial with zero constant term; every polynomial with zero constant term is in the ideal $(x, y)$.

**Exercise 8.3.** Part (1) is a almost identical to an example discussed in lecture. A similar proof works for part (2), noticing that $\mathbb{k}[x]$ is also a PID. For part (3), you need the result in Exercise 8.2 (4). It is easy to show $(x, y)$ is prime. To show it is maximal, assume $(x, y) \subseteq I \subseteq \mathbb{k}[x, y]$. We have either $I = (x, y)$, or $I$ contains a polynomial with non-zero constant term. In the second possibility, you need to explain why that non-zero constant term, when considered as a constant polynomial, is also in $I$. Explain why it is a unit in the ring $\mathbb{k}[x, y]$, and why it implies $I = \mathbb{k}[x, y]$. The ideal $(x)$ can easily proved to be prime. It is not maximal because we can find an ideal strictly between $(x)$ and $\mathbb{k}[x, y]$. What is this intermediate ideal?

**Exercise 8.4.** You are already given step-by-step hints for this problem. For the last step in part (1), you can use Exercise 8.2 (2). For part (2), you need to use the definition of an ideal to show that $J$ is an ideal, i.e., the sum of two elements in $J$ is still in $J$; the product of any element in $J$ and any element in $\mathcal{O}_K$ is still in $J$.

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