MA40238 NUMBER THEORY 2013/14 SEMESTER 1 WEEK 5 OVERVIEW

RECAP FROM LAST WEEK

We talked about

- Legendre and Jacobi symbols (their similarities and differences);
- their properties (7 rules for each);
- computation of Legendre and Jacobi symbols;
- given an integer a, how to find all odd primes for which a is a quadratic residue.

Monday Lecture

The topic is *Gauss' Lemma*, which is

- an essential ingredient in the proof of Quadratic Reciprocity;
- another handy tool for computing Legendre symbols.

We will see

- the statement of Gauss' Lemma;
- how to use it to compute Legendre symbols (in examples);
- the proof of Gauss' Lemma;
- a consequence of Gauss' Lemma.

TUESDAY LECTURE

There are two topics:

- We will prove Quadratic Reciprocity, using a nice geometric observation due to Gauss.
- We will prove a refinement of Euclid's Theorem on infinitely many primes: there are infinitely many primes congruent to 1 modulo 4 (or −1 modulo 4). You are expected to learn the method and prove similar statements.

Date: October 27, 2014.