

## MA40238 NUMBER THEORY 2013/14 SEMESTER 1 WEEK 6 OVERVIEW

### RECAP

In last two weeks, we talked about: (i) quadratic residues and Legendre symbols; (ii) Jacobi symbols (which can be used to greatly simplify the calculation of Legendre symbols); (iii) a proof of the quadratic reciprocity.

### MONDAY LECTURE

We study algebraic number theory for the rest of the semester. This lecture focuses on the notions of an *algebraic number* and an *algebraic integer*.

You need to understand their definitions.

There are three methods for proving a certain complex number is an algebraic integer.

- by definition; i.e. explicitly write down a polynomial as required in the definition;
- a criterion which allows us to consider it as an eigenvalue of a square matrix;
- the sums and products of algebraic integers are also algebraic integers.

But be aware that these criteria are not very helpful for proving a certain complex number is *not* an algebraic integer. (The definition can be used to do that in some simple cases.) We will see how to deal with this problem next week.

### TUESDAY LECTURE

This lecture focuses on two notions.

- The first notion is a *number field*. You need to understand: (i) its definition; (ii) typical examples; (iii) its property.
- The second notion is the *trace* and *norm* of an element in a number field. You need to understand: (i) their definitions; (ii) how to compute them in examples and remember the results for quadratic fields; (iii) their properties.