MA40238 NUMBER THEORY 2013/14 SEMESTER 1 WEEK 9 OVERVIEW

RECAP

Last week, we discussed

(1) The norm of an ideal (3 descriptions).

(2) The ascending chain condition.

(3) The unique factorisation of ideals.

Monday Lecture

Topic. What is the ideal class group and the class number? Why are we interested?

(1) We will define three related notions: ideal class, ideal class group, and class number. Roughly speaking, for each number field K, we can associate to it a group, called the *ideal class group*. We will see that it is always a finite abelian group. Every element in this group is called an *ideal class* (as you can tell from the name). The order of the group (i.e., the number of elements in this group) is called the *class number*.

(2) We will explain why they are important. Roughly speaking, the class number measures how far the ring \mathcal{O}_K is from being a PID.

(3) We will also learn a few geometric notions, which will be used in the statement of the Minkowski's Theorem.

TUESDAY LECTURE

Topic. Minkowski's Theorem.

(1) We will state and prove this theorem. It is quite a surprising result for two reasons: one can hardly imagine why the condition and the conclusion are related at all by intuition; one can hardly imagine why it could be helpful in algebraic number theory.

(2) We will prepare a scenario for a nice application of the Minkowski's Theorem that we will see next week. The main topic for next week will be the computation of class numbers based on this setup.

Date: November 22, 2014.