Math 19 Calculus **Winter 2007** **Instructor: Geir Helleloid**

Final Exam
Friday, March 24

Name: ______________________________

I agree to abide by the honor code:

Signature: ____________________________

- You have 3 hours (8:30 – 11:30).
- No notes, books, or calculators are permitted.
- **You must show all work to receive credit!**
- Please check your solutions carefully.

1. ________ (/50 points)
2. ________ (/20 points)
3. ________ (/20 points)
4. ________ (/20 points)
5. ________ (/20 points)
6. ________ (/20 points)

7. __________ (/10 points)

Total. __________ (/160 points)

1. (a) (10 points) Let

$$y = \frac{2}{x-1} - \frac{1}{\sqrt{x}}.$$

Find dy/dx.

(b) (10 points) Let

$$y = (\sin x)^{\cos x}.$$

Find dy/dx. Your answer should be a function of x only.

(c) (10 points) Let

$$y = \sqrt{\tan(x^2)}.$$

Find dy/dx.

(d) (10 points) Find the equation of the tangent line to the curve

$$e^{x^2} + e^{y^2} = 2e$$

at the point $(-1, 1)$.

(e) (10 points) Let

$$y = \frac{(2x+1)^4 \sin(x^2)}{(\ln x)\sqrt{3x-1}}.$$

Find $\frac{dy}{dx}$. Your answer should be a function of x only.

2. (20 points) Let

$$f(x) = \ln(x^2 - 1).$$

(a) (10 points) You must show all your work, but please write your final answers in the box.

The domain of $f(x)$ is:	________________
$f(x)$ is increasing on:	________________
$f(x)$ is decreasing on:	________________
$f(x)$ has local maxima at:	________________
$f(x)$ has local minima at:	________________
$f(x)$ is concave up on:	________________
$f(x)$ is concave down on:	________________

(b) (4 points) Compute the following four limits.

$$\lim_{x \to \infty} \ln(x^2 - 1)$$

$$\lim_{x \to -\infty} \ln(x^2 - 1)$$

$$\lim_{x \to 1^+} \ln(x^2 - 1)$$

$$\lim_{x \to -1^-} \ln(x^2 - 1)$$

(c) (1 point) List all vertical and horizontal asymptotes of $y = \ln(x^2 - 1)$.

(d) (5 points) Using your answers from parts (a) and (b), sketch a graph of

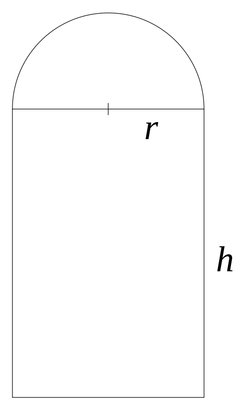
$$f(x) = \ln\left(x^2 - 1\right).$$

Even if your answers in parts (a) and (b) are wrong, if your sketch correctly uses those answers, you may earn partial credit.

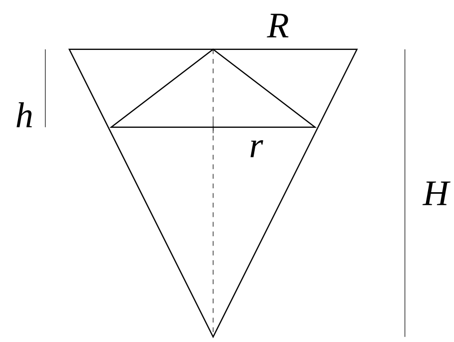
3. (20 points) A particle is moving along the curve $x^2 - 4xy - y^2 = -5$. Given that the x-coordinate of the particle is changing at 3 units/second, *how fast is the distance from the particle to the origin changing* when the particle is at the point $(1, 2)$? Hint: As an intermediate step, you should compute the value of $\frac{dy}{dt}$ when $x = 1$ and $y = 2$.

4. (20 points) A balloon is rising at a constant speed of 1 m/sec. A girl is cycling along a straight road at a speed of 2 m/sec. When she passes under the balloon it is 3 m above her. How fast is the distance between the girl and the balloon increasing 2 seconds later?

5. (20 points) A Norman window consists of a rectangle surmounted by a semicircle, as shown. Given that the total area of the window is $A = 8+2\pi$, find the minimum possible perimeter of the window. (Please note the horizontal line between the rectangle and the semicircle does not count as part of the perimeter.) Hint: The total area has been carefully chosen so that the minimum perimeter occurs at a very simple value of r. If your optimal value of r is complicated, you have done something incorrectly.



6. (20 points) Suppose you have a cone with *constant* height H and *constant* radius R, and you want to put a smaller cone "upside down" inside the larger cone (see the picture). If h is the height of the smaller cone, what should h be to maximize the volume of the smaller cone? The optimal value of h will depend on H. Recall that the volume of a cone with base radius r and height h is given by the formula $V = \frac{1}{3}\,\pi r^2 h$.



7. (10 points) For parts (a) and (b), compute the given limits, if they exist. If you assert that a limit does not exist, you need to justify your answer to get full credit.

(a) (5 points)

$$\lim_{x \to \infty} (\sqrt{x^2 - 3x + 1} - \sqrt{x^2 + 2})$$

(b) (5 points)

$$\lim_{x \to 2} e^{\frac{1}{x-2}}$$