Math 19 Calculus **Summer 2008**

Final Exam
Monday, August 11

Name: ______________________________

I agree to abide by the honor code:

Signature: ____________________________

- You have 3 hours.
- No notes, books, or calculators are permitted.
- **You must show all work to receive credit!**
- Please check your solutions carefully.

1. ________ (/50 points)
2. ________ (/20 points)
3. ________ (/30 points)
4. ________ (/20 points)
5. ________ (/20 points)
6. ________ (/20 points)
7. ________ (/20 points)
8. ________ (/10 points)
9. ________ (/5 points)

Total. ________ (/195 points)

1. (a) (10 points) Let

$$y = \frac{2}{x-1} - \frac{x+2}{\sqrt{x}}.$$

Find dy/dx.

(b) (10 points) Let

$$y = (\sin 2x)^x.$$

Find dy/dx. Your answer should be a function of x only.

(c) (10 points) Let

$$y = \ln\left(\tan\left(x^2\right)\right).$$

Find dy/dx.

(d) (10 points) Find the equation of the tangent line to the curve

$$e^{x^2} + e^{y^2} = 2e$$

at the point $(-1, 1)$.

(e) (10 points) Let

$$y = \frac{(2x+1)^4 \cos\left(x^2\right)}{(\ln x)\sqrt{3x-1}}.$$

Find $\frac{dy}{dx}$. Your answer should be a function of x only.

2. (20 points) Let

$$f(x) = x^3 - 3x + 1.$$

(a) (10 points) You must show all your work, but please write your final answers in the box.

The domain of $f(x)$ is:	____________________
$f(x)$ is increasing on:	____________________
$f(x)$ is decreasing on:	____________________
$f(x)$ has local maxima at:	____________________
$f(x)$ has local minima at:	____________________
$f(x)$ is concave up on:	____________________
$f(x)$ is concave down on:	____________________

(b) (5 points) Give the number of zeros (roots) of $f(x) = x^3 - 3x + 1$. Justify your answer. (Hint: One possible solution involves computing $f(-2), f(-1), f(1), f(2)$ and using that f is a continuous function.)

(c) (5 points) Using all of the above, sketch a graph of

$$f(x) = x^3 - 3x + 1.$$

Even if your answers in parts (a) and (b) are wrong, if your sketch correctly uses those answers, you may earn partial credit.

3. (30 points) Let

$$f(x) = \frac{x^2+1}{x^2-1}.$$

(a) (10 points) You must show all your work, but please write your final answers in the box.

The domain of $f(x)$ is:	______________
$f(x)$ is increasing on:	______________
$f(x)$ is decreasing on:	______________
$f(x)$ has local maxima at:	______________
$f(x)$ has local minima at:	______________
$f(x)$ is concave up on:	______________
$f(x)$ is concave down on:	______________

(b) (15 points) Compute the following six limits.

i. $\displaystyle\lim_{x\to\infty} \frac{x^2+1}{x^2-1}$

ii. $\displaystyle\lim_{x\to-\infty} \frac{x^2+1}{x^2-1}$

iii. $\displaystyle\lim_{x\to1^-} \frac{x^2+1}{x^2-1}$

iv. $\displaystyle\lim_{x\to1^+} \frac{x^2+1}{x^2-1}$

v. $\displaystyle\lim_{x\to-1^-} \frac{x^2+1}{x^2-1}$

vi. $\displaystyle\lim_{x\to-1^+} \frac{x^2+1}{x^2-1}$

(c) (5 points) Sketch a graph of

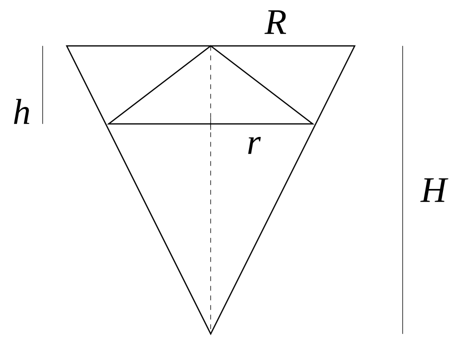
$$f(x) = \frac{x^2+1}{x^2-1}.$$

Even if your answers in parts (a) and (b) are wrong, if your sketch correctly uses those answers, you may earn partial credit.

4. (20 points) A particle is moving along the curve $x^2 - 4xy - y^2 = -5$. Given that the x-coordinate of the particle is changing at 3 units/second, *how fast is the distance from the particle to the origin changing* when the particle is at the point $(1, 2)$? Hint: As an intermediate step, you should compute the value of $\frac{dy}{dt}$ when $x = 1$ and $y = 2$.

5. (20 points) A balloon is rising at a constant speed of 1 m/sec. A girl is cycling along a straight road at a speed of 2 m/sec. When she passes under the balloon it is 3 m above her. How fast is the distance between the girl and the balloon increasing 2 seconds later?

6. (20 points) Suppose you have a cone with *constant* height $H = 3$ and *constant* radius $R = 1$, and you want to put a smaller cone "upside down" inside the larger cone (see the picture). If h is the height of the smaller cone, what should h be to maximize the volume of the smaller cone? The optimal value of h will depend on H. Recall that the volume of a cone with base radius r and height h is given by the formula $V = \frac{1}{3}\,\pi r^2 h$. (Hint: Use similar triangles to get the relationship between h and r.)



7. (20 points) Compute the given limits, if they exist. If you assert that a limit does not exist, you need to justify your answer to get full credit.

   (a) (5 points)

$$\lim_{x\to\infty} (\sqrt{x^2 - 3x + 1} - \sqrt{x^2 + 2})$$

   (b) (5 points)

$$\lim_{x\to 2} e^{\frac{1}{x-2}}$$

(c) (5 points)

$$\lim_{x \to \infty} \frac{x^3 + 1}{e^x}$$

(d) (5 points)

$$\lim_{x \to 2^+} \frac{2 - x}{\ln(x - 2)}$$

8. (10 points) Estimate the following using linear approximation.

(a) (5 points)

$$\sin^2(\pi + .01)$$

(b) (5 points)

$$f(x) = x^3 - 2x + 1$$

Estimate f(-.05).

9. (5 points) – Do not attempt this question until completing the rest of the exam.

   (a) State the intermediate value theorem.

   (b) State the mean value theorem.

   (c) On July 4, 2008, Joey Chestnut ate 59 hot-dogs in 10 minutes to win Nathan's Hot Dog Eating Contest. Assume that the number of hot dogs that he consumes is a differentiable function of time and that at at the start of the competition, his rate of comsumption is 0 hot dogs per minute. Give a precise argument that he was eating at a rate of 4 hot dogs per minute at some point during the competition.