

# Midterm

## Wednesday, July 23

Name: \_\_\_\_\_

I agree to abide by the honor code:

Signature: \_\_\_\_\_

- You have 2 hours (7:00 – 9:00).
- No notes, books, or calculators are permitted.
- **You must show all work to receive credit!**
- Please only use techniques and theorems studied in this class.
- Please check your solutions carefully.

1. \_\_\_\_\_ (/10 points)

2. \_\_\_\_\_ (/10 points)

3. \_\_\_\_\_ (/10 points)

4. \_\_\_\_\_ (/10 points)

5. \_\_\_\_\_ (/10 points)

6. \_\_\_\_\_ (/10 points)

7. \_\_\_\_\_ (/10 points)

8. \_\_\_\_\_ (/10 points)

9. \_\_\_\_\_ (/10 points)

10. \_\_\_\_\_ (/10 points)

Total. \_\_\_\_\_ (/100 points)

1. Answer the following questions with true or false. No explanation is needed.

(a) The product rule states that  $(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$ .

(b) The quotient rule states that  $\frac{d}{dx} \left( \frac{f(x)}{g(x)} \right) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$ .

(c)  $x < 0$  or  $x > 1$  is written in interval notation as  $(-\infty, 0] \cup [1, \infty)$ .

(d) If the derivative of a function  $f(x)$  is negative on some interval  $(a, b)$ , then  $f(x)$  is decreasing on the interval  $(a, b)$ .

(e) All continuous functions are differentiable at all points in their domains.

(f) The  $\lim_{x \rightarrow \infty} \sin x$  exists and equals some real number  $L$ .

(g) If  $\lim_{x \rightarrow a} f(x)$  and  $\lim_{x \rightarrow a} g(x)$  both exist as finite limits, then  $\lim_{x \rightarrow a} (f(x)g(x))$  must exist, and  $\lim_{x \rightarrow a} (f(x)g(x)) = [\lim_{x \rightarrow a} f(x)][\lim_{x \rightarrow a} g(x)]$ .

(h) The second derivative of  $f(x) = cx$  where  $c$  is a constant, always equals 0.

(i) The  $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2}$  does not exist because  $\frac{0}{0}$  is an indeterminate form.

(j) The  $n$ th derivative of  $f(x) = e^x$  is  $e^x$ .

2. (a) Compute

$$\lim_{x \rightarrow \infty} \arctan(\ln x)$$

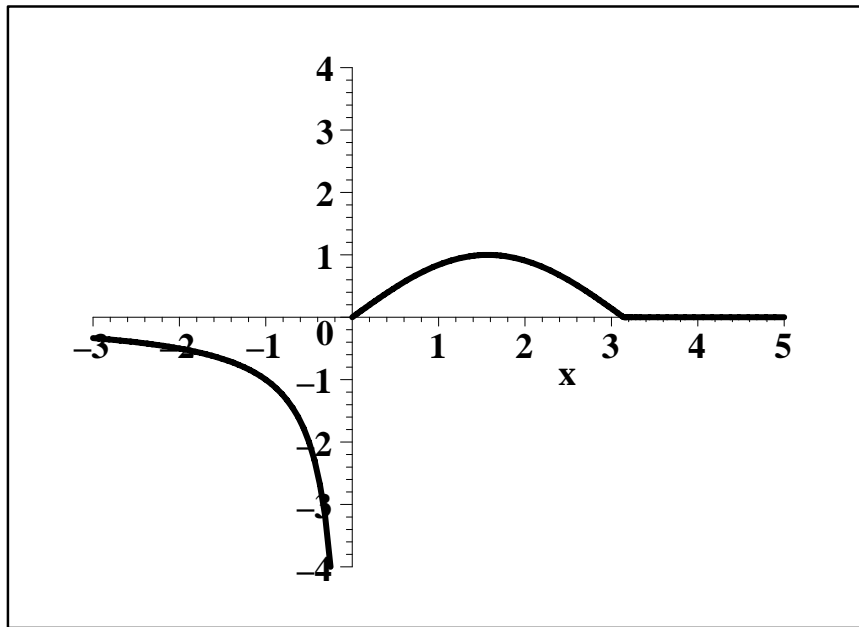
(b) Compute

$$\lim_{x \rightarrow 4} \frac{x - \sqrt{3x + 4}}{4 - x}.$$

(c) Compute

$$\lim_{x \rightarrow \infty} \frac{5x^3 + 9x - 4}{\sqrt{4x^6 - x^3 + 8}}.$$

3. Given the following function  $f(x)$



(a) Compute the following limits

$$\lim_{x \rightarrow \infty} f(x)$$

$$\lim_{x \rightarrow 0^-} f(x)$$

$$\lim_{x \rightarrow 0^+} f(x)$$

$$\lim_{x \rightarrow 0} f(x)$$

$$\lim_{x \rightarrow -\infty} f(x)$$

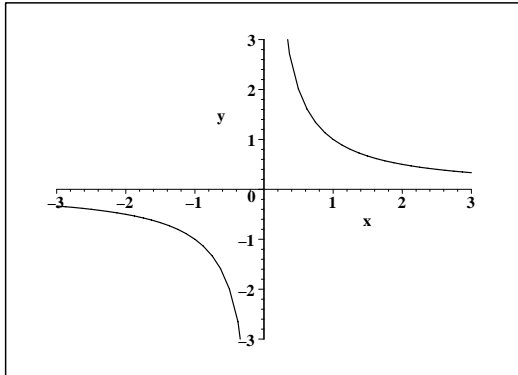
(b) State the numbers at which  $f(x)$  is not differentiable.

4. Let  $f(x) = \frac{1}{x}$ .

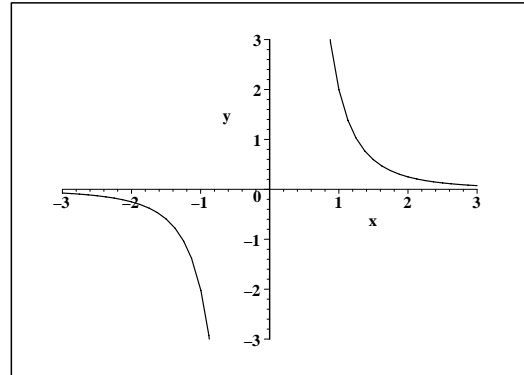
(a) Using the limit definition of the derivative, find  $f'(x)$ .

(b) Check your answer by calculating the derivative in another way: state and apply a differentiation rule.

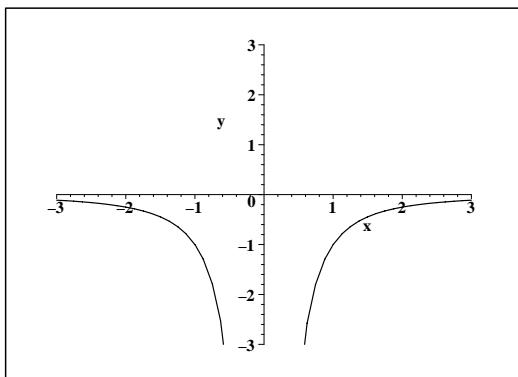
5. The graphs of three functions are drawn in the left column. The graphs of their derivatives are drawn in the right column. Match each function with its derivative. You do not need to justify your answer.



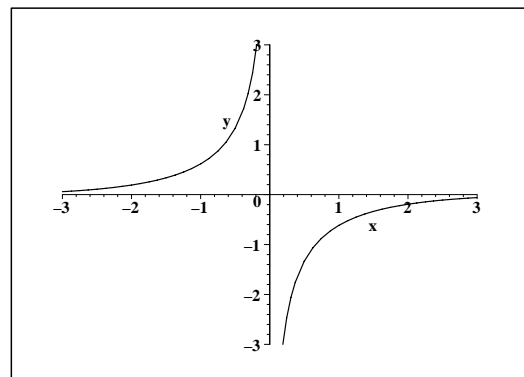
(A)



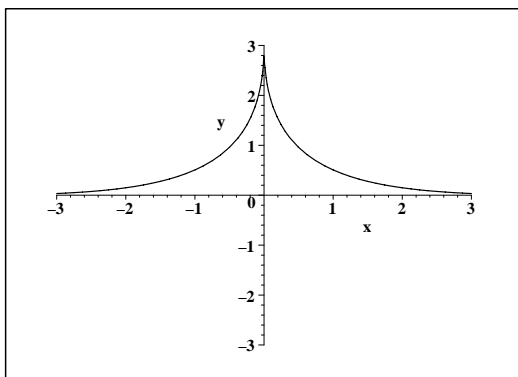
(a)



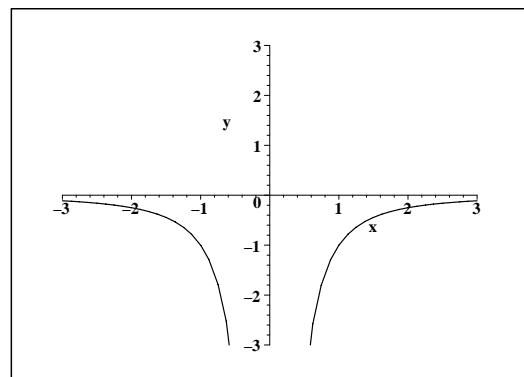
(B)



(b)



(C)



(c)

6. The derivative of a function  $f(x)$  is defined as a limit of certain quotients.

(a) Write down a limit definition of  $f'(x)$

(b) Let  $f(x) = x^2$ . Graph  $f(x)$  and explain what  $f'(1)$  represents and what the numbers  $\frac{f(1+h)-f(1)}{h}$  represent geometrically.

- (c) Graph an example of a continuous function that is *not* differentiable at  $x = 0$  and give a geometric argument why the derivative fails to exist at that point.



7. Calculate  $f'(x)$  when:

(a)  $f(x) = \sin(x) \tan(x)$

(b)  $f(x) = e^{2x} \cos(2x) + 10x^5 \sin(x)$

8. Let  $f(x) = \frac{e^x}{x+1}$

(a) Calculate  $f'(x)$ .

(b) Calculate  $f''(x)$ .

9. Let  $y = \sqrt{5 - 2x^2}$ .

(a) Calculate  $\frac{dy}{dx}$ .

(b) Calculate  $\frac{d}{dx} \cos(y(x))$ .

10. (a) Complete the definition:  $f(x)$  is continuous at  $x = a$  provided the following conditions hold:

(b) Let  $f(x) = \begin{cases} x^{\frac{1}{3}} \cos \frac{1}{x^2} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ .

Is  $f(x)$  continuous at 0? Give a rigorous justification for your answer.