PRACTICE MIDTERM

1. True/False Questions. No explanation is needed.

(1) $f(x) = |x - 2|$ is one-to-one.

(2) $\lim_{x\to 5}\left(\frac{2x}{x-5} - \frac{10}{x-5}\right) = \lim_{x\to 5}\frac{2x}{x-5} - \lim_{x\to 5}\frac{10}{x-5}$

(3) A function can have infinitely many horizontal asymptotes.

(4) If f is continuous on $[0, 2]$, then f is differentiable on $[0, 2]$.

(5) The n-th derivative of $f(x) = e^{2x}$ is $2^n e^{2x}$.

2. The graph of $f(x)$ is shown. Answer the following questions and explain your reasoning:

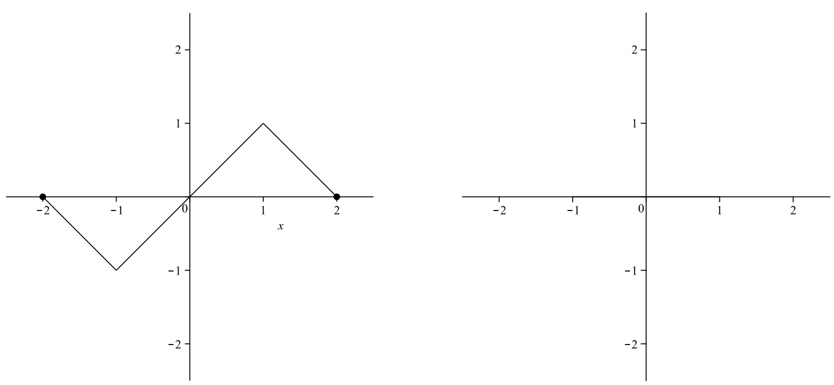
(1) What is the domain of f?

(2) What is the range of f?

(3) Is f one-to-one?

(4) Where is f not differentiable?

(5) Sketch the graph of $-f(-x) + 1$ on the coordinate system.



3. Evaluate the following limits or show they do not exist.

(1) $\lim_{x\to -1}\frac{x^2 - 3x - 4}{x + 1}$

(2) $\lim_{x\to\frac{1}{2}} \ln(\sin(\pi x))$

(3) $\lim_{x\to 2}(x^2-4)^2 \sin\left(\frac{1}{x-2}\right)$

(4) $\lim_{x\to\infty} \frac{3-x}{x^2-3x+2}$

(5) $\lim_{x\to 0} f(x)$ where

$$f(x) = \begin{cases} e^x & \text{if } x < 0, \\ 0 & \text{if } x = 0, \\ \tan^2 x + 1 & \text{if } x > 0 \end{cases}$$

4. Let $g(t) = \frac{t+3}{t-1}$.

(1) Find the equation(s) of all vertical asymptote(s) of g.
(2) Find the equation(s) of all horizontal asymptotes of g.
(3) Find $g^{-1}(t)$.

5. Show there exists a number a between $[0, \frac{\pi}{2}]$ such that the graph of $x^2 - \sin x$ has a horizontal tangent line at a.

6. Using the limit definition of the derivative, compute the derivative of $f(x) = 2\sqrt{x}$. What is the equation of the tangent line to the curve when $x = 1$?

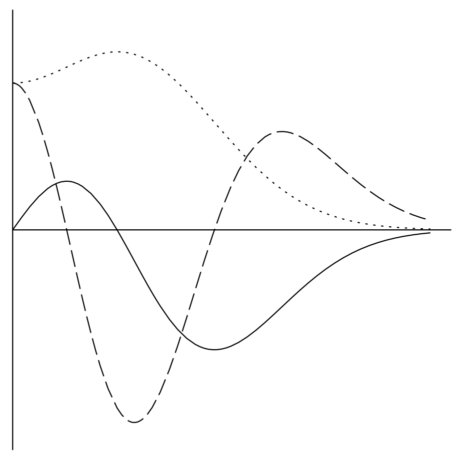
7. Find the derivatives of the following functions:

(1) $f(x) = x^5 - x^{3/4} + 1$
(2) $f(x) = x \ln x$
(3) $f(x) = \sin(2e^x)$
(4) $f(x) = \frac{x^2-1}{x^2+1}$
(5) $f(x) = \ln\left(\frac{\sqrt{x}\cot x}{e^x}\right)$
(6) $f(x) = |x|$

8. The displacement (in centimeters) of a particle moving back and forth along a straight line is given by the equation of motion $s = 2^t + t^3 + 1$ where t is measured in seconds.

(1) Find the average velocity of the particle during $[1, 3]$.
(2) Find the velocity of the particle at $t = 1$.
(3) Find the acceleration of the particle at $t = 1$.

9. The figure shows the graphs of f, f' and f''. Identify each curve and explain your choices.



10. Sketch a possible graph of $f(x)$ which satisfies all the conditions:

(i) $f(0) = 1$, (ii) $\lim_{x\to-\infty} f(x) = 0$, (iii) $f'(0) = 1$, (iv) f is increasing on $[-1, 1]$,
(v) f is concave downward on $(-\infty, -1)$, (vi) f is concave upward on $(0, 1]$,
(vii) $\lim_{x\to3^-} f(x) = 5$, (viii) $\lim_{x\to3^+} f(x) = 2$, (ix) f is decreasing on $[3, \infty)$,
(x) $\lim_{x\to\infty} f(x) = -\infty$.