Math 19: Calculus **Summer 2009**

Homework 1- Solutions

Homework scores are out of 30 points. Ten problems were graded each worth 3 points. The graded problems from this homework set are 2 and 34 from section 1.1, 12 from section 1.2, 6, 20, and 48 from section 1.3, 30 from section 1.5, and 22, 40, and 56 from section 1.6.

Please check that your solutions are correct on the ungraded problems.

Section 1.1

2.

(a) The point $(-4, -2)$ is on the graph of f, so $f(-4) = -2$. The point $(3, 4)$ is on the graph of g, so $g(3) = 4$.

(b) We are looking for the values of x for which the y-values are equal. The y-values for f and g are equal at the points $(-2, 1)$ and $(2, 2)$, so the desired values of x are -2 and 2.

(c) $f(x) = -1$ is equivalent to $y = -1$. When $y = -1$, we have $x = -3$ and $x = 4$.

(d) As x increases from 0 to 4, y decreases from 3 to -1. Thus, f is decreasing on the interval $[0, 4]$.

(e) The domain of f consists of all x-values on the graph of f. For this function, the domain is $-4 \leq x \leq 4$, or $[-4, 4]$. The range of f consists of all y-values on the graph of f. For this function, the range is $-2 \leq y \leq 3$, or $[-2, 3]$.

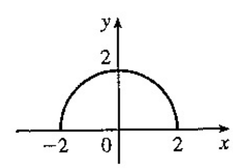
(f) The domain of g is $[-4, 3]$ and the range is $[0.5, 4]$.

8.

No, the curve is not the graph of a function since for $x = 0$, ± 1, and ± 2, there are infinitely many points on the curve.

34.

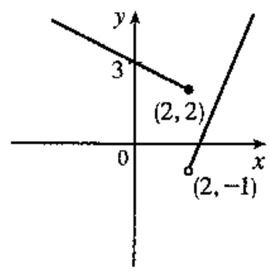
$h(x) = \sqrt{4 - x^2}$. Now $y = \sqrt{4 - x^2} \quad \Rightarrow \quad y^2 = 4 - x^2 \quad \Leftrightarrow \quad x^2 + y^2 = 4$, so the graph is the top half of a circle of radius 2 with center at the origin. The domain is $\{x \mid 4 - x^2 \geq 0\} = \{x \mid 4 \geq x^2\} = \{x \mid 2 \geq |x|\} = [-2, 2]$. From the graph, the range is $0 \leq y \leq 2$, or $[0, 2]$.



44.

42. $f(x) = \begin{cases} 3 - \frac{1}{2}x & \text{if } x \leq 2 \\ 2x - 5 & \text{if } x > 2 \end{cases}$

The domain is $\mathbb{R}$.



Section 1.2

4.

(a) The graph of $y = 3x$ is a line (choice G).

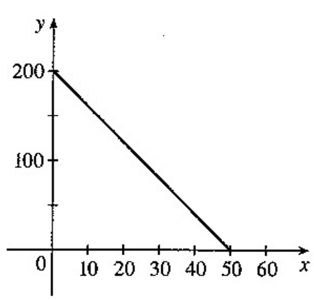
(b) $y = 3^x$ is an exponential function (choice f).

(c) $y = x^3$ is an odd polynomial function or power function (choice F).

(d) $y = \sqrt[3]{x} = x^{1/3}$ is a root function (choice g).

12.

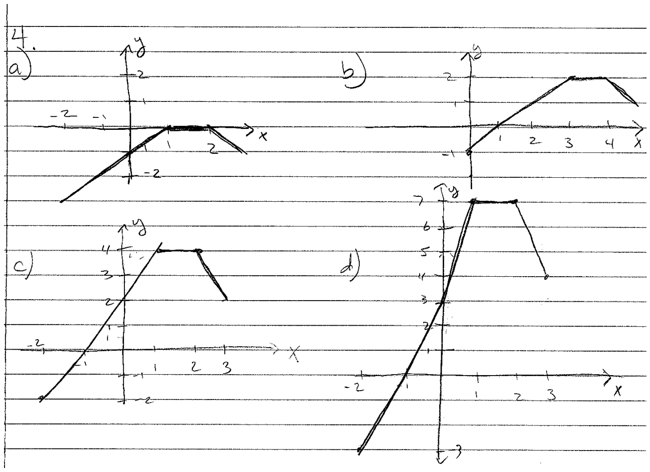
(a)



(b) The slope of -4 means that for each increase of 1 dollar for a rental space, the number of spaces rented *decreases* by 4. The y-intercept of 200 is the number of spaces that would be occupied if there were no charge for each space. The x-intercept of 50 is the smallest rental fee that results in no spaces rented.

Section 1.3

4.



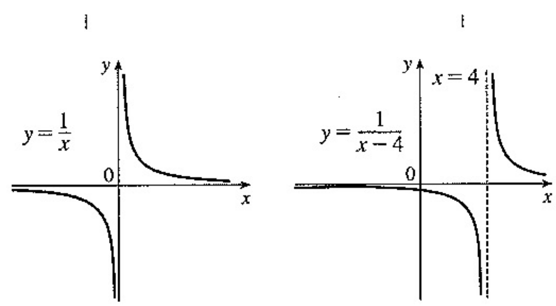
6.

The graph of $y = f(x) = \sqrt{3x - x^2}$ has been shifted 2 units to the right and stretched vertically by a factor of 2. Thus, a function describing the graph is

$$y = 2f(x-2) = 2\sqrt{3(x-2)-(x-2)^2} = 2\sqrt{3x-6-(x^2-4x+4)} = 2\sqrt{-x^2+7x-10}$$

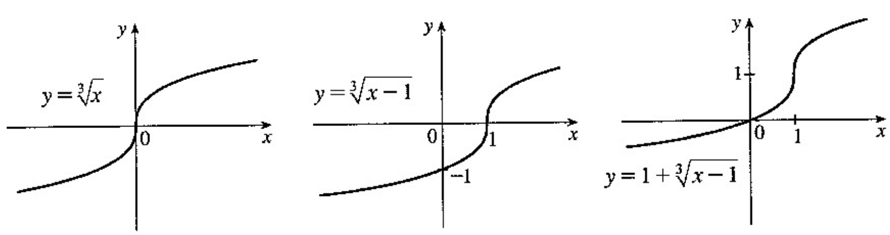
16.

. $y = 1/(x-4)$: Start with the graph of $y = 1/x$ and shift 4 units to the right.



20.

. $y = 1 + \sqrt[3]{x-1}$: Start with the graph of $y = \sqrt[3]{x}$, shift 1 unit to the right, and then shift 1 unit upward.



36.

a) $f \circ g = \dfrac{sin(2x)}{1 + sin(2x)}$

The domain of $sin(2x)$ is $\mathbb{R}$ so the numerator and denominators are defined everywhere.
The function is not defined when the $1 + sin(2x) = 0 \Rightarrow sin(2x) = -1 \Rightarrow$
$2x = 3\pi n/4 \Rightarrow x = 3\pi n/8$ where n is an integer.
The domain of $f \circ g$ is all real numbers except $\{x = 3\pi n/8$ where n is an integer$\}$.

b) $g \circ f = sin\left(\dfrac{2x}{1+x}\right)$

Again $sin(x)$ is defined everywhere so we only need to check the function $\frac{2x}{1+x}$.
This function is defined everywhere except $x = -1$. Thus the domain of $g \circ f$ is
$\mathbb{R} \setminus \{x = -1\}$

c) $f \circ f = \dfrac{\frac{x}{1+x}}{1 + \frac{x}{1+x}} = \dfrac{x}{1 + x + x} = \dfrac{x}{1 + 2x}$

This function is defined everywhere except where $1 + 2x = 0 \Rightarrow x = -1/2$.
The domain of $f \circ f$ is $\mathbb{R} \setminus \{x = -1/2\}$

d) $g \circ g = sin(2sin(2x))$

Since $sin(x)$ is defined on $\mathbb{R}$ then this function is also defined on $\mathbb{R}$.

46.

Let $g(t) = \tan t$ and $f(t) = \dfrac{t}{1+t}$. Then $(f \circ g)(t) = f(g(t)) = f(\tan t) = \dfrac{\tan t}{1 + \tan t} = u(t)$.

48.

Let $h(x) = |x|$, $g(x) = 2 + x$, and $f(x) = \sqrt[8]{x}$. Then
$(f \circ g \circ h)(x) = f(g(h(x))) = f(g(|x|)) = f(2 + |x|) = \sqrt[8]{2 + |x|} = H(x)$.

Section 1.5

18.

(a) This reflection consists of first reflecting the graph about the x-axis (giving the graph with equation $y = -e^x$) and then shifting this graph $2 \cdot 4 = 8$ units upward. So the equation is $y = -e^x + 8$.

(b) This reflection consists of first reflecting the graph about the y-axis (giving the graph with equation $y = e^{-x}$) and then shifting this graph $2 \cdot 2 = 4$ units to the right. So the equation is $y = e^{-(x-4)}$.

20.

(a) The sine and exponential functions have domain $\mathbb{R}$, so $g(t) = \sin(e^{-t})$ also has domain $\mathbb{R}$.

(b) The function $g(t) = \sqrt{1 - 2^t}$ has domain $\{t \mid 1 - 2^t \geq 0\} = \{t \mid 2^t \leq 1\} = \{t \mid t \leq 0\} = (-\infty, 0]$.

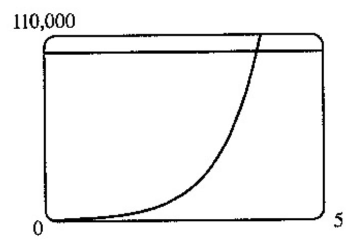
30.

(a) Three hours represents 6 doublingperiods (one doubling period is 30 minutes). $500 \cdot 2^6 = 32{,}000$

(b) In t hours, there will be $2t$ doubling periods. The initial population is 500, so the population y at time t is $y = 500 \cdot 2^{2t}$.

(c) $t = \frac{40}{60} = \frac{2}{3} \quad \Rightarrow \quad y = 500 \cdot 2^{2(2/3)} \approx 1260$

(d) We graph $y_1 = 500 \cdot 2^{2t}$ and $y_2 = 100{,}000$. The two curves intersect at $t \approx 3.82$, so the population reaches 100,000 in about 3.82 hours.



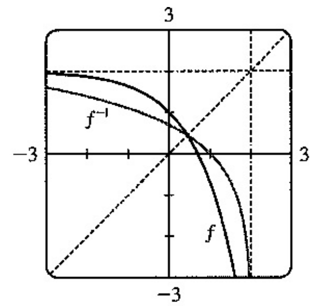
Section 1.6

22.

$y = f(x) = \dfrac{4x-1}{2x+3} \quad \Rightarrow \quad y(2x+3) = 4x-1 \quad \Rightarrow \quad 2xy + 3y = 4x - 1 \quad \Rightarrow \quad 3y + 1 = 4x - 2xy \quad \Rightarrow$
$3y + 1 = (4-2y)x \quad \Rightarrow \quad x = \dfrac{3y+1}{4-2y}$. Interchange x and y: $y = \dfrac{3x+1}{4-2x}$. So $f^{-1}(x) = \dfrac{3x+1}{4-2x}$.

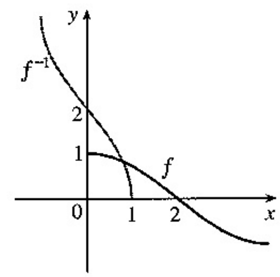
28.

$y = f(x) = 2 - e^x \quad \Rightarrow \quad e^x = 2 - y \quad \Rightarrow \quad x = \ln(2-y)$. Interchange x and y: $y = \ln(2-x)$. So $f^{-1}(x) = \ln(2-x)$. From the graph, we see that f and f^{-1} are reflections about the line $y = x$.



30.

Reflect the graph of f about the line $y = x$.



40. $\ln(a+b) + \ln(a-b) - 2\ln(c)$
$= \ln((a+b)(a-b)) - 2\ln(c)$
$= \ln((a+b)(a-b)) - \ln(c^2)$
$= \ln(\dfrac{(a+b)(a-b)}{c^2})$
$= \ln(\dfrac{a^2-b^2}{c^2})$

56.

(a) For $f(x) = \ln(2 + \ln x)$, we must have $2 + \ln x > 0 \quad \Rightarrow \quad \ln x > -2 \quad \Rightarrow \quad x > e^{-2}$. Thus, the domain of f is (e^{-2}, ∞).

(b) $y = f(x) = \ln(2 + \ln x) \quad \Rightarrow \quad e^y = 2 + \ln x \quad \Rightarrow \quad \ln x = e^y - 2 \quad \Rightarrow \quad x = e^{e^y - 2}$. Interchange x and y: $y = e^{e^x - 2}$. So $f^{-1}(x) = e^{e^x - 2}$. The domain of f^{-1}, as well as the range of f, is $\mathbb{R}$.