Math 19: Calculus **Summer 2009**

Homework 2- Solutions

Homework scores are out of 30 points. Ten problems were graded each worth 3 points. The graded problems from this homework set are 8 and 14 from section 2.2, 18, 32, and 38 from section 2.3, 32 and 42 from section 2.4, and 4, 26, and 42 from section 2.5.

Please check that your solutions are correct on the ungraded problems.

Section 2.2

6.

a) $\lim_{x \to -3^-} h(x) = 4$

b) $\lim_{x \to -3^+} h(x) = 4$

c) $\lim_{x \to -3} h(x) = 4$ because the left and right-hand limits agree.

d) $h(-3)$ does not exist since it $h(x)$ is not defined at $x = -3$

e) $\lim_{x \to 0^-} h(x) = 1$

f) $\lim_{x \to 0^+} h(x) = -1$

g) $\lim_{x \to 0} h(x)$ does not exist because the left and right hand limits are not the same.

h) $h(0) = 1$

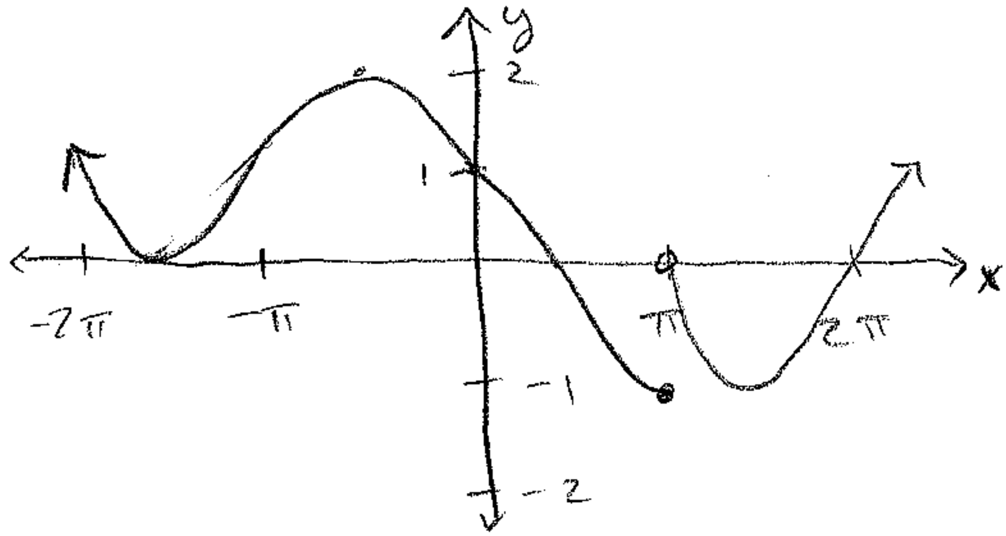
i) $\lim_{x \to 2} h(x) = 2$

j) $h(2)$ does not exist since the function is not defined here.

k) $\lim_{x \to 5^+} h(x) = 3$

l) $\lim_{x \to 5^-} h(x)$ does not exist because there are infinitely many values of x that approach 5 from the left for which $h(x) = 3$ and infinitely many values of x that approach 5 from the left for which $h(x) = 2$.

8.



As you can see from the graph, when $0 < a < \pi$, and $a > \pi$, $\lim_{x \to a} f(x)$ exists since the left and right-hand limits are the same.
The limit does not exist at $a = \pi$ since the left and right-hand limits are not the same.

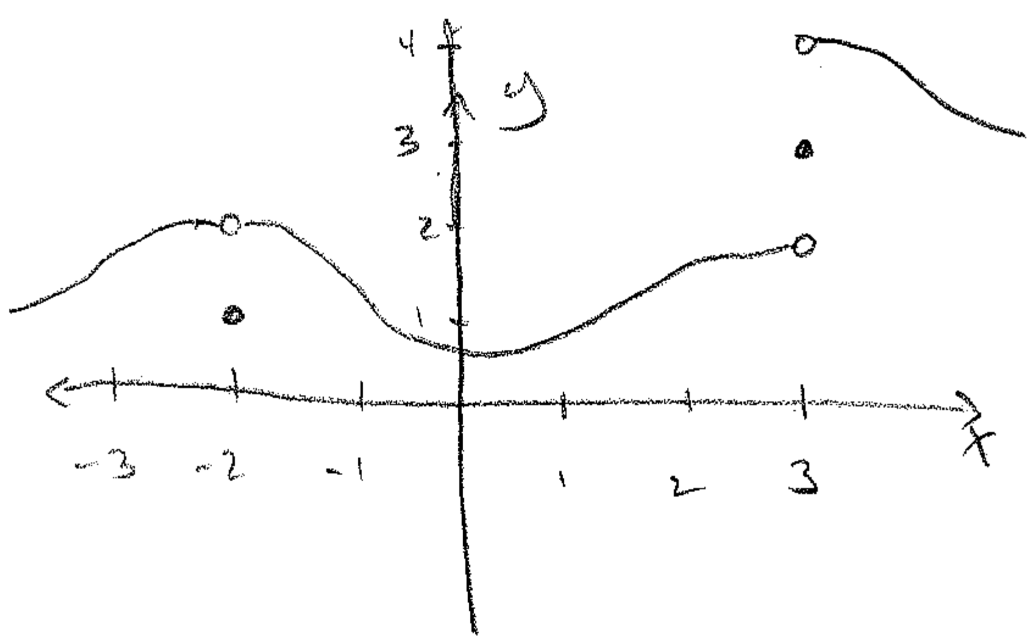
12. (sorry this is crooked)

-1

$\lim_{t \to 12^-} f(t) = 150$ mg and $\lim_{t \to 12^+} f(t) = 300$ mg. These limits show that there is an abrupt change in the amount of drug in the patient's bloodstream at $t = 12$ h. The left-hand limit represents the amount of the drug just before the fourth injection. The right-hand limit represents the amount of the drug just after the fourth injection.

10. $\lim f(x) = 1$, $\lim f(x) = -1$, $\lim_{x \to 2^-} f(x) = 0$,

14.



Section 2.3

2.

(a) $\lim_{x\to 2}[f(x)+g(x)] = \lim_{x\to 2} f(x) + \lim_{x\to 2} g(x) = 2 + 0 = 2$

(b) $\lim_{x\to 1} g(x)$ does not exist since its left- and right-hand limits are not equal, so the given limit does not exist.

(c) $\lim_{x\to 0}[f(x)g(x)] = \lim_{x\to 0} f(x) \cdot \lim_{x\to 0} g(x) = 0 \cdot 1.3 = 0$

(d) Since $\lim_{x\to -1} g(x) = 0$ and g is in the denominator, but $\lim_{x\to -1} f(x) = -1 \neq 0$, the given limit does not exist.

(e) $\lim_{x\to 2} x^3 f(x) = \left[\lim_{x\to 2} x^3\right]\left[\lim_{x\to 2} f(x)\right] = 2^3 \cdot 2 = 16$

(f) $\lim_{x\to 1}\sqrt{3+f(x)} = \sqrt{3+\lim_{x\to 1} f(x)} = \sqrt{3+1} = 2$

10.

$$\lim_{x\to 4}\frac{x^2-4x}{x^2-3x-4} = \lim_{x\to 4}\frac{x(x-4)}{(x-4)(x+1)} = \lim_{x\to 4}\frac{x}{x+1} = \frac{4}{4+1} = \frac{4}{5}$$

18.

$$\lim_{h\to 0}\frac{\sqrt{1+h}-1}{h} = \lim_{h\to 0}\frac{\sqrt{1+h}-1}{h}\cdot\frac{\sqrt{1+h}+1}{\sqrt{1+h}+1} = \lim_{h\to 0}\frac{(1+h)-1}{h\left(\sqrt{1+h}+1\right)} = \lim_{h\to 0}\frac{h}{h\left(\sqrt{1+h}+1\right)}$$

$$= \lim_{h\to 0}\frac{1}{\sqrt{1+h}+1} = \frac{1}{\sqrt{1}+1} = \frac{1}{2}$$

22.

$$\lim_{t\to 0}\left(\frac{1}{t}-\frac{1}{t^2+t}\right) = \lim_{t\to 0}\frac{(t^2+t)-t}{t(t^2+t)} = \lim_{t\to 0}\frac{t^2}{t\cdot t(t+1)} = \lim_{t\to 0}\frac{1}{t+1} = \frac{1}{0+1} = 1$$

32.

$-1 \le \sin(\pi/x) \le 1 \quad\Rightarrow\quad e^{-1} \le e^{\sin(\pi/x)} \le e^1 \quad\Rightarrow\quad \sqrt{x}/e \le \sqrt{x}\,e^{\sin(\pi/x)} \le \sqrt{x}\,e$. Since $\lim_{x\to 0^+}(\sqrt{x}/e) = 0$ and $\lim_{x\to 0^+}(\sqrt{x}\,e) = 0$, we have $\lim_{x\to 0^+}\left[\sqrt{x}\,e^{\sin(\pi/x)}\right] = 0$ by the Squeeze Theorem.

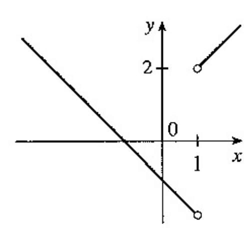
38.

(a) (i) $\lim_{x\to 1^+}\frac{x^2-1}{|x-1|} = \lim_{x\to 1^+}\frac{x^2-1}{x-1} = \lim_{x\to 1^+}(x+1) = 2$

(ii) $\lim_{x\to 1^-}\frac{x^2-1}{|x-1|} = \lim_{x\to 1^-}\frac{x^2-1}{-(x-1)} = \lim_{x\to 1^-}-(x+1) = -2$

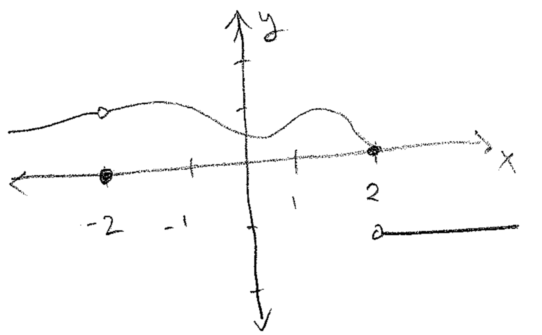
(b) No, $\lim_{x\to 1} F(x)$ does not exist since $\lim_{x\to 1^+} F(x) \neq \lim_{x\to 1^-} F(x)$.

(c)



Section 2.4

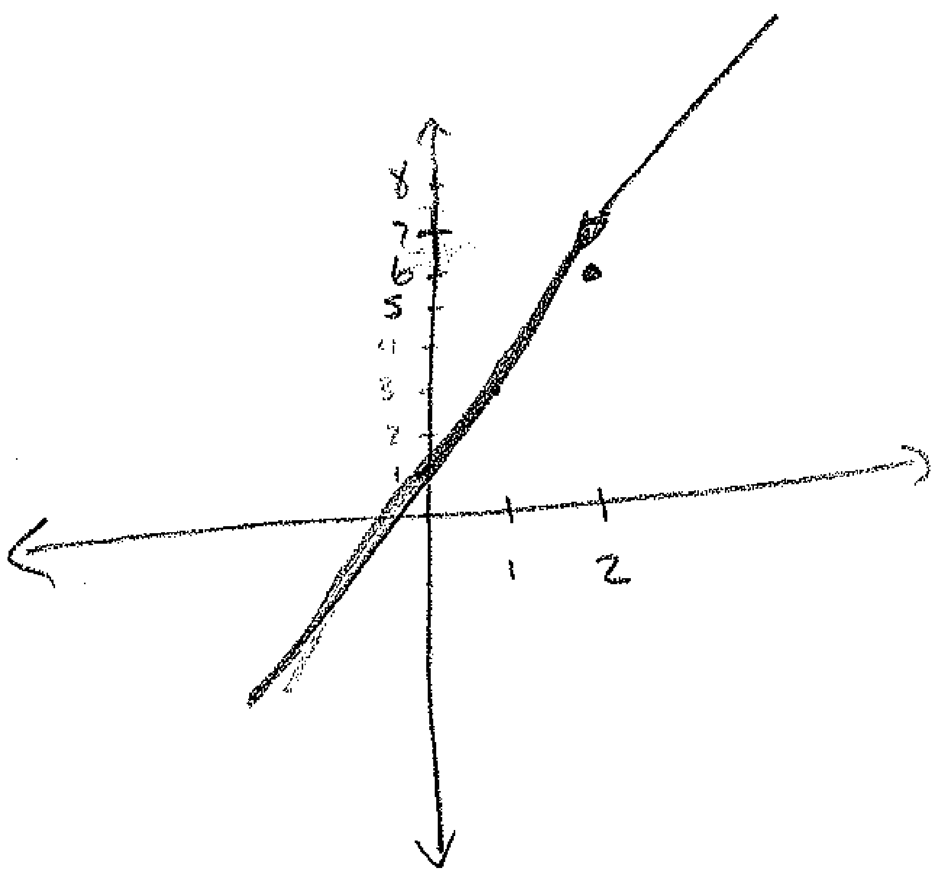
8.



18.

$$\lim_{x \to 3} f(x) = \lim_{x \to 3} \frac{2x^2 - 5x - 3}{x - 3} = \lim_{x \to 3} 2x + 1 = 2(3) + 1 = 7$$

Since the limit as $x \to 3$ does not equal $f(3) = 6$ this function is not continuous at $x = 3$.



32.

By Theorem 5, each piece of F is continuous on its domain. We need to check for continuity at $r = R$.

$\lim\limits_{r\to R^-} F(r) = \lim\limits_{r\to R^-} \dfrac{GMr}{R^3} = \dfrac{GM}{R^2}$ and $\lim\limits_{r\to R^+} F(r) = \lim\limits_{r\to R^+} \dfrac{GM}{r^2} = \dfrac{GM}{R^2}$, so $\lim\limits_{r\to R} F(r) = \dfrac{GM}{R^2}$. Since $F(R) = \dfrac{GM}{R^2}$, F is continuous at R. Therefore, F is a continuous function of r.

36.

Note: there is an error in this problem.
If we assume that $f(x) = ax^2 - bx + 3$ at $x = 2$ as well then the problem is doable.
Making the above assumption:
Without any work we know that $f(x)$ is continuous everywhere except $x = 2$ and $x = 3$.

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} \frac{x^2-4}{x-2} = \lim_{x\to 2^-} x+2 = 4$$
$$\lim_{x\to 2^+} f(x) = \lim x \to 2^+ ax^2 - bx + 3 = a(4) - b(2) + 3 = f(2)$$

To make $f(x)$ continuous at $x = 2$ we need $4a - 2b + 3 = 4$.

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} ax^2 - bx + 3 = 9a - 3b + 3$$
$$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} 2x - a + b = 2(3) - a + b = f(3)$$

To make $f(x)$ continuous at $x = 3$ we need 9a-3b+3=6-a+b.
Solve these two equations to find: $a = b = 1/2$.

42.

$f(x) = \sqrt[3]{x} + x - 1$ is continuous on the interval $[0, 1]$, $f(0) = -1$, and $f(1) = 1$. Since $-1 < 0 < 1$, there is a number c in $(0, 1)$ such that $f(c) = 0$ by the Intermediate Value Theorem. Thus, there is a root of the equation $\sqrt[3]{x} + x - 1 = 0$, or $\sqrt[3]{x} = 1 - x$, in the interval $(0, 1)$.

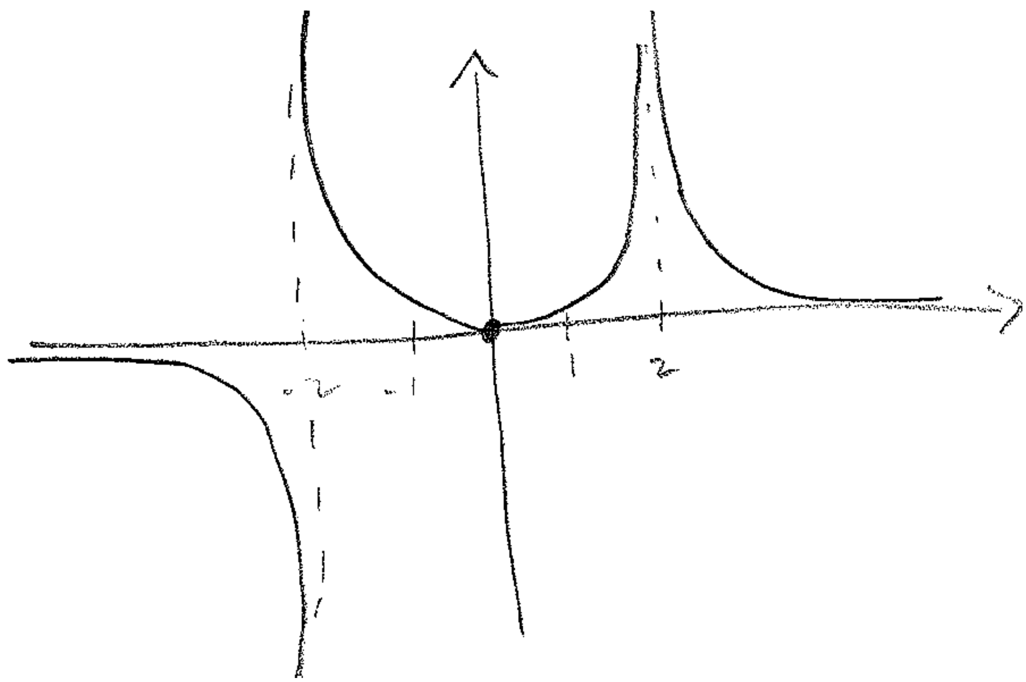
Section 2.5

4.

(a) $\lim_{x\to\infty} g(x) = 2$ (b) $\lim_{x\to-\infty} g(x) = -2$ (c) $\lim_{x\to 3} g(x) = \infty$

(d) $\lim_{x\to 0} g(x) = -\infty$ (e) $\lim_{x\to -2^+} g(x) = -\infty$ (f) Vertical: $x = -2$, $x = 0$, $x = 3$; Horizontal: $y = -2$, $y = 2$

6.



20.

$$\lim_{xto2^-} \frac{x^2-2x}{x^2-4x+4} = \lim_{x\to 2^-} \frac{x}{x-2}$$

When x is close to but slightly less than 2, $x > 0$ and $x - 2 < 0$ so $\frac{x}{x-2} < 0$. Thus the limit is $-\infty$.

24.

$$\lim_{t\to-\infty} \frac{t^2+2}{t^3+t^2-1} = \lim_{t\to-\infty} \frac{(t^2+2)/t^3}{(t^3+t^2-1)/t^3} = \lim_{t\to-\infty} \frac{1/t+2/t^3}{1+1/t-1/t^3} = \frac{0+0}{1+0-0} = 0$$

26.

$$\begin{aligned}
\lim_{x\to\infty} \left(\sqrt{x^2+ax}-\sqrt{x^2+bx}\right) &= \lim_{x\to\infty} \frac{\left(\sqrt{x^2+ax}-\sqrt{x^2+bx}\right)\left(\sqrt{x^2+ax}+\sqrt{x^2+bx}\right)}{\sqrt{x^2+ax}+\sqrt{x^2+bx}} \\
&= \lim_{x\to\infty} \frac{(x^2+ax)-(x^2+bx)}{\sqrt{x^2+ax}+\sqrt{x^2+bx}} = \lim_{x\to\infty} \frac{[(a-b)x]/x}{\left(\sqrt{x^2+ax}+\sqrt{x^2+bx}\right)/\sqrt{x^2}} \\
&= \lim_{x\to\infty} \frac{a-b}{\sqrt{1+a/x}+\sqrt{1+b/x}} = \frac{a-b}{\sqrt{1+0}+\sqrt{1+0}} = \frac{a-b}{2}
\end{aligned}$$

36.

If we let $t = \tan x$, then as $x \to (\pi/2)^+$, $t \to -\infty$. Thus, $\lim\limits_{x \to (\pi/2)^+} e^{\tan x} = \lim\limits_{t \to -\infty} e^t = 0$.

42.

$\lim\limits_{x\to\infty} \dfrac{2e^x}{e^x - 5} = \lim\limits_{x\to\infty} \dfrac{2e^x}{e^x - 5} \cdot \dfrac{1/e^x}{1/e^x} = \lim\limits_{x\to\infty} \dfrac{2}{1-(5/e^x)} = \dfrac{2}{1-0} = 2$, so $y = 2$ is a horizontal asymptote.

$\lim\limits_{x\to-\infty} \dfrac{2e^x}{e^x - 5} = \dfrac{2(0)}{0-5} = 0$, so $y = 0$ is a horizontal asymptote. The denominator is zero (and the numerator isn't) when

$e^x - 5 = 0 \quad \Rightarrow \quad e^x = 5 \quad \Rightarrow \quad x = \ln 5$. $\lim\limits_{x\to(\ln 5)^+} \dfrac{2e^x}{e^x - 5} = \infty$ since the numerator approaches 10 and the denominator approaches 0 through positive values as $x \to (\ln 5)^+$. Similarly,

$\lim\limits_{x\to(\ln 5)^-} \dfrac{2e^x}{e^x - 5} = -\infty$. Thus, $x = \ln 5$ is a vertical asymptote.

The graph confirms our work.