

Midterm Review – Thursday, July 16

1. Find the domain of the function

$$h(u) = \frac{\sqrt{5-u}}{\sqrt[4]{u^2-3u}}.$$

2. Consider the following function

$$g(x) = \begin{cases} x^2 - 2 & \text{if } x < 0; \\ -1 & \text{if } x = 0; \\ 2x - 2 & \text{if } 0 < x \leq 1; \\ e^{x-1} & \text{if } x > 1. \end{cases}$$

- (a) For what values a does $\lim_{x \rightarrow a} g(x)$ not exist?
(b) For what values a is the function $g(x)$ discontinuous at $x = a$?
(c) Sketch the graph of $g(x)$.
3. Is there a number a such that the limit

$$\lim_{x \rightarrow -2} \frac{3x^2 + ax + 2}{x^2 + x - 2}$$

exists? If so, find the value a and the value of the limit.

4. Find the limit

$$\lim_{x \rightarrow \infty} (\sqrt{9x^2 + x} - 3x).$$

5. Show that the equation

$$2^x = \sqrt{4 - x^2}$$

has a root in the interval $[0, 1]$.

6. Use any method to differentiate the function

$$f(x) = \frac{xe^{\sin x}}{\sqrt{x+1}}.$$

7. Let

$$f(x) = \frac{1+x}{1-x}.$$

- (a) Use the limit definition of the derivative to calculate $f'(x)$.
(b) Find a formula for $f^{-1}(x)$.