

## Quiz 3 Solutions

1. (2 points) Let  $f(x)$  be a function. Write down a formula which defines its derivative at the number  $a$ .

*Solution.*

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} \quad \text{or} \quad f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}.$$

2. (2 points) Let  $f(x) = 2x$ . Use the definition in the previous problem to compute  $f'(1)$  (the derivative of  $f(x)$  at the number 1). Show your calculations.

*Solution.*

$$f'(1) = \lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \rightarrow 0} \frac{2(1+h) - 2}{h} = \lim_{h \rightarrow 0} \frac{2h}{h} = \lim_{h \rightarrow 0} 2 = 2.$$

or

$$f'(1) = \lim_{x \rightarrow 1} \frac{f(x) - f(1)}{x - 1} = \lim_{x \rightarrow 1} \frac{2x - 2}{x - 1} = \lim_{x \rightarrow 1} \frac{2(x - 1)}{x - 1} = \lim_{x \rightarrow 1} 2 = 2.$$

3. (2 points) Use your answer in the previous problem to find the equation of the tangent line to  $f(x)$  at the point  $(1, 2)$ .

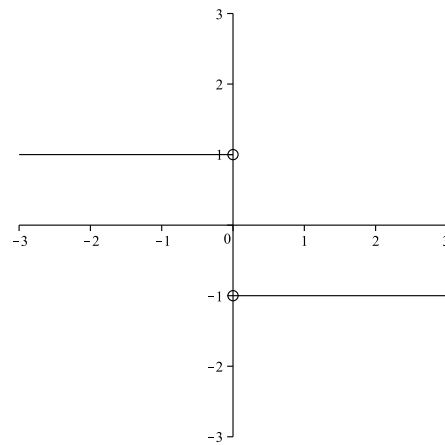
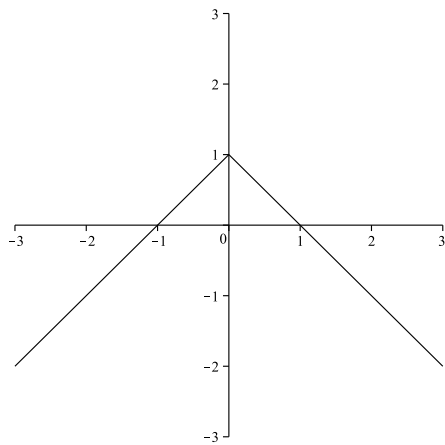
*Solution.* From previous problem we know the slope of the tangent line at the point  $(1, 2)$  is 2. By the point-slope formula, the equation of the line is

$$y - 2 = 2(x - 1)$$

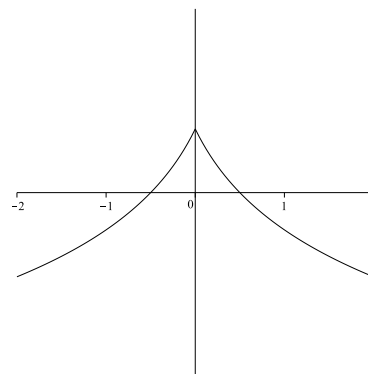
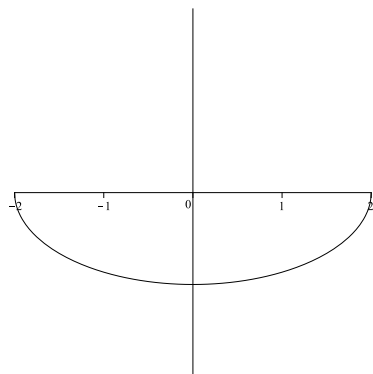
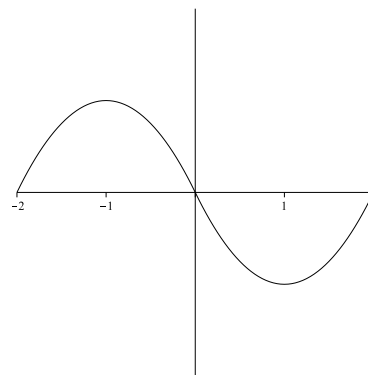
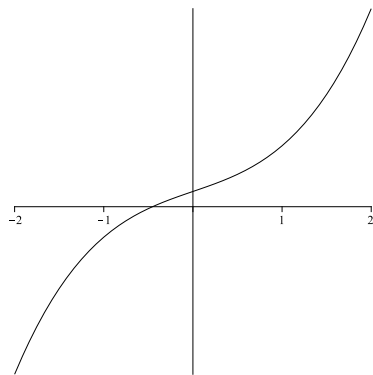
which simplifies to

$$y = 2x.$$

4. (2 points) On the left is the graph of a function  $f(x)$ . Sketch the graph of its derivative  $f'(x)$  in the coordinate system on the right.



5. (2 points) On the interval  $(0, 2)$ , which of the following functions are concave upward? Please circle all correct answers. No justification is necessary. (Hint: there might be more than one correct answer.)



*Solution.* All of the above graphs are correct answers.