Quiz 4 Solutions

1. *(2 points)* What is the derivative of $\arctan x$?

   Solution.
   $$\frac{1}{1+x^2}$$

2. *(2 points)* $y = \ln\left(\frac{x}{x+1}\right)$, find $\frac{dy}{dx}$.

   Solution.
   $$y = \ln\left(\frac{x}{x+1}\right) = \ln x - \ln(x+1).$$
   Therefore by differentiating both sides we can obtain:
   $$y' = \frac{1}{x} - \frac{1}{x+1}.$$

3. *(2 points)* Suppose x, y satisfy an equation $y^2 + y = x$. Find the relation of y', x, y by differentiating both sides of the equation.

   Solution. By implicit differentiation,
   $$\begin{aligned}\frac{d}{dx}(y^2+y) &= \frac{d}{dx}x\\ 2yy' + y' &= 1\\ y'(2y+1) &= 1\\ y' &= \frac{1}{2y+1}\end{aligned}$$

4. *(2 points)* Let x, y be described in previous problem. Find y' at the point $(2, 1)$.

   Solution. Because from last problem we know $y' = \frac{1}{2y+1}$. Plug in $y = 1$ and we can get $y' = \frac{1}{3}$.

5. *(2 points)* Find the linear approximation $L(x)$ of $f(x) = \sqrt{x}$ at the point $(1, 1)$.

   Solution. The linear approximation of $f(x)$ at a is $L(x) = f(a) + f'(a)(x - a)$. Since it's given $a = 1$, $f(x) = \sqrt{x}$, we can find $f'(x) = \dfrac{1}{2\sqrt{x}}$. Thus $f(a) = 1$, $f'(a) = \frac{1}{2}$, and

$$L(x) = 1 + \frac{1}{2}(x - 1) = \frac{1}{2}(x + 1)\,.$$