Quiz 6 Solutions

1. *(2 points)* L'Hospital Rule states if f and g are differentiable and $g'(x) \neq 0$ near a (except possibly at a). And suppose that

$$\lim_{x\to a} f(x) = 0 \quad \text{and} \quad \lim_{x\to a} g(x) = 0$$
$$\text{or that} \quad \lim_{x\to a} f(x) = \pm\infty \quad \text{and} \quad \lim_{x\to a} g(x) = \pm\infty\,.$$

Then

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \underline{\hspace{3cm}}$$

Solution.

$$\lim_{x\to a} \frac{f'(x)}{g'(x)}$$

2. *(2 points)* Find

$$\lim_{x\to 1} \frac{x^3-1}{x^2-1}.$$

Solution. First method:

$$\lim_{x\to 1} \frac{x^3-1}{x^2-1} = \lim_{x\to 1} \frac{(x-1)(x^2+x+1)}{(x-1)(x+1)} = \lim_{x\to 1} \frac{x^2+x+1}{x+1} = \frac{3}{2}.$$

Second method. Because it's of the type 0/0, we can use L'Hospital Rule.

$$\lim_{x\to 1} \frac{x^3-1}{x^2-1} = \lim_{x\to 1} \frac{3x^2}{2x} = \frac{3}{2}\,.$$

3. *(2 points)* Suppose f is a differentiable function on $[-2, 2]$ and $f(-2) = 1$, $f(2) = -3$. Prove there is a number c between -2 and 2 such that $f'(c) = -1$.

Solution. Because f is a differentiable function on $[-2, 2]$ and

$$\frac{f(2) - f(-2)}{2 - (-2)} = \frac{-4}{4} = -1\,,$$

By the Mean Value Theorem there is a c between -2 and 2 such that $f'(c) = -1$.

4. *(4 points)* Find the point on the line $x + y = 6$ that is closest to the point $(6, 8)$.

Solution. Suppose (x, y) is one the line $x + y = 6$, then $y = 6 - x$. The distance between (x, y) and $(6, 8)$ is

$$\sqrt{(x-6)^2 + (y-8)^2} = \sqrt{(x-6)^2 + (6-x-8)^2} = \sqrt{(x-6)^2 + (-2-x)^2}$$

Because minimizing the distance is equivalent to minimizing the square of the distance, we can let $f(x) = (x-6)^2 + (-2-x)^2$ and find the minimum of f.

$$f'(x) = 2(x-6) + 2(-2-x)(-1) = 2x - 12 + 2x + 4 = 4x - 8$$

If $f'(x) = 0$, $x = 2$, $y = 6 - 2 = 4$. $f''(x) = 4 > 0$ for all x. By Second Derivative Test for Absolute Extrema, $(2, 4)$ is the closet point to $(6, 8)$.